More Branch and Bound Algorithms

Algorithmic Problems Around the Web #3

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Outline

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1. Variations of Metric Trees
2. M-Trees

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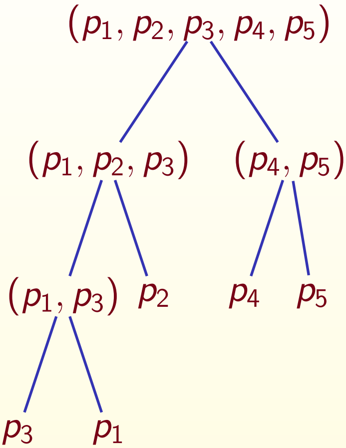
Part I

Variations of Metric Trees

Branch and Bound: Range Search

Task: find all $i \quad d(p_i, q) \leq r$:

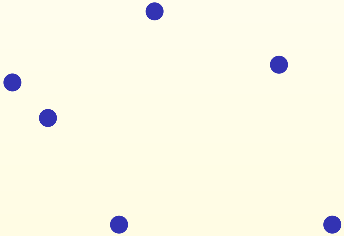
1. Make a depth-first traversal of search hierarchy
2. At every node compute the lower bound for its subtree
3. Prune branches with lower bounds above r



Vantage-Point Partitioning

Uhlmann'91, Yianilos'93:

1. Choose some object p in database (called pivot)
2. Choose partitioning radius r_p
3. Put all p_i such that $d(p_i, p) \leq r$ into "inner" part, others to the "outer" part
4. Recursively repeat



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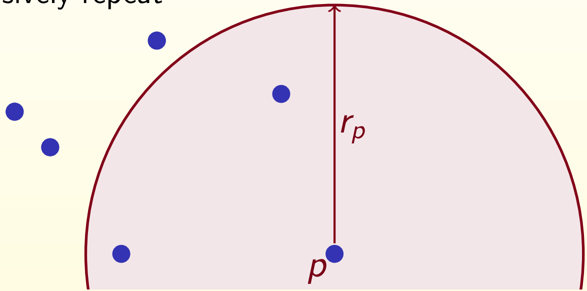
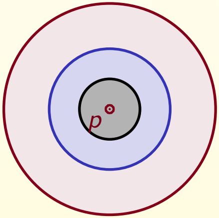


Variations of Vantage-Point Trees

- **Burkhard-Keller tree:** pivot used to divide the space into m rings Burkhard&Keller'73
- **MVP-tree:** use the same pivot for different nodes in one level Bozkaya&Ozsoyoglu'97
- **Post-office tree:** use $r_p + \delta$ for inner branch, $r_p - \delta$ for outer branch McNutt'72



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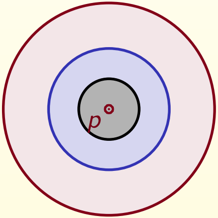

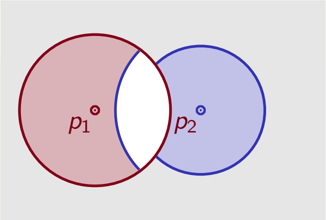

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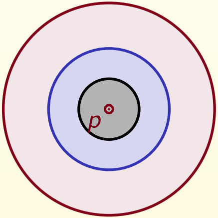

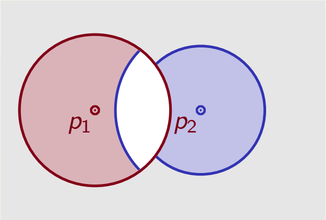

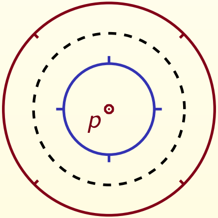

Generalized Hyperplane Tree

Partitioning technique (Uhlmann'91):

- Pick two objects (called pivots) p_1 and p_2
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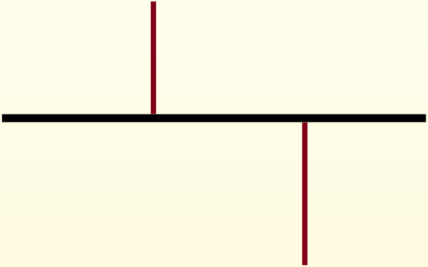

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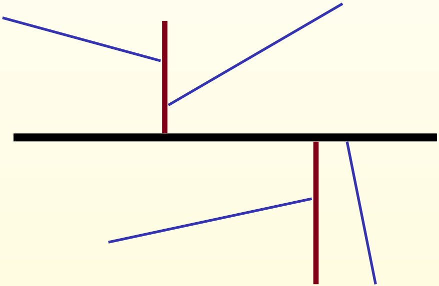
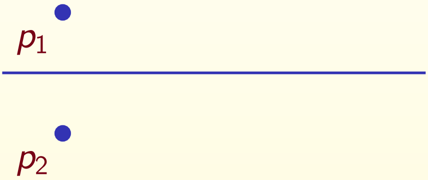


GH-Tree: Pruning Conditions

For r-range search:

If $d(q, p_1) > d(q, p_2) + 2r$ prune the left branch
If $d(q, p_1) < d(q, p_2) - 2r$ prune the right branch

For $|d(q, p_1) - d(q, p_2)| \leq 2r$ we have to inspect both branches



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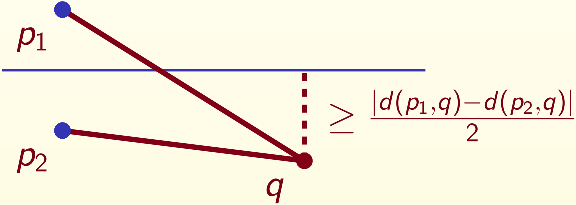
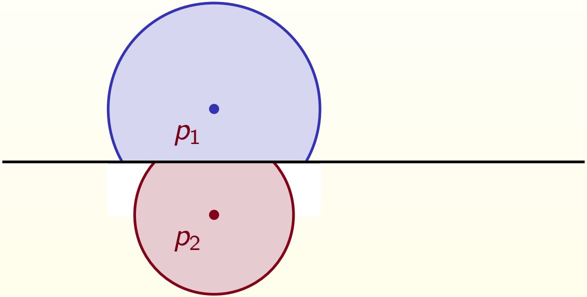


Bisector trees

Let's keep the covering radius for p_1 and left branch, for p_2 and right branch: useful information for stronger pruning conditions



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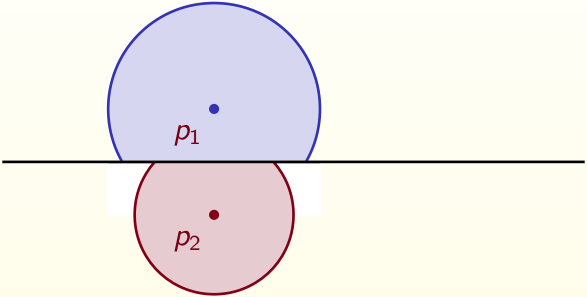

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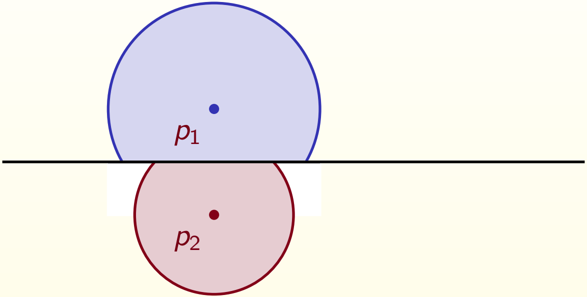

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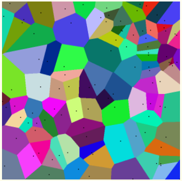
Exercise: prove that covering radii are monotonically decrease in mb-trees

Geometric Near-Neighbor Access Tree

Brin'95:

- Use m pivots
- Branch i consists of objects for which p_i is the closest pivot
- Stores minimal and maximal distances from pivots to all "brother"-branches



Part II

M-trees

M-tree: Data structure

Ciaccia, Patella, Zezula'97:

- All database objects are stored in leaf nodes (buckets of fixed size)
- Every internal nodes has associated pivot, covering radius and legal range for number of children (e.g. 2-3)
- Usual depth-first or best-first search

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Special algorithms for insertions and deletions a-la B-tree

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    2. Both pivots are added to the node’s parent, which may cause it to be split, and so on

Part III

k-d Trees, R-trees

Advantages of Euclidean Space

- Rich mathematical formalisms for defining a boundary of any set

  Examples: rectangles, hyperplanes, polynomial curves

- Easy computation of lower bound on distance between query point and any set boundary

- Easy definable mappings to smaller spaces

k-d Tree

Preprocessing: Bentley, 1975

Top-down partitioning

On level l: split the current set

by hyperplane orthogonal to $l \bmod k$ axis

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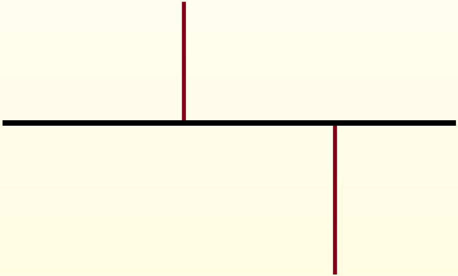

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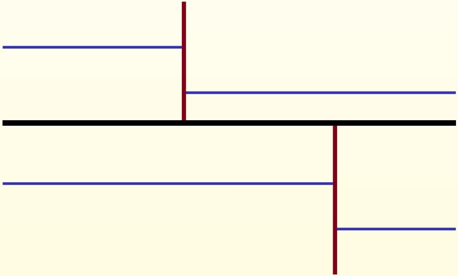

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Keep bounding rectangles
Every time: merge current rectangles
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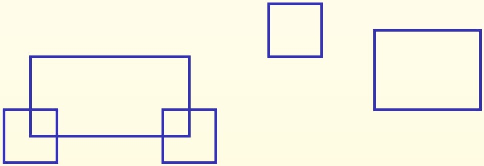

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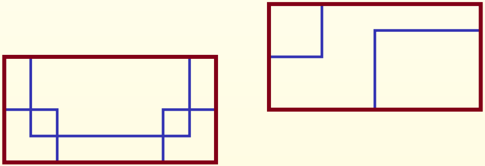

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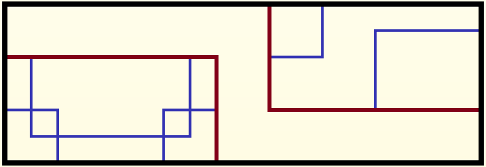

Thanks for your attention! Questions?

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