Combinatorial Framework for Similarity Search

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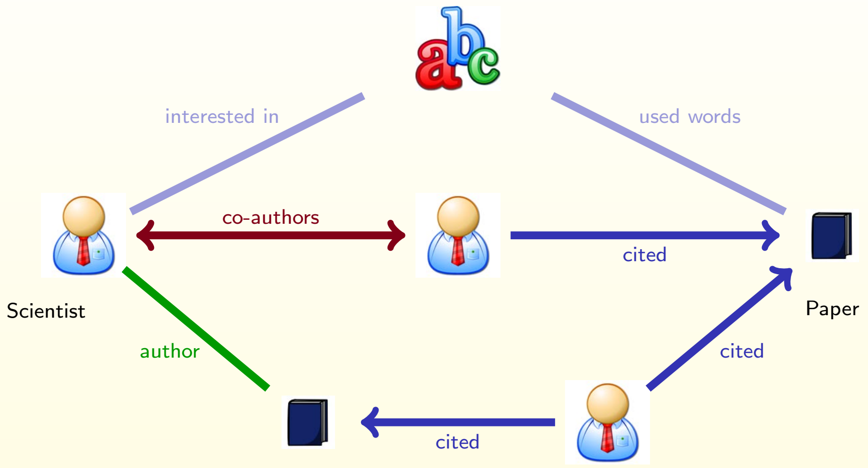
ESSCASS 2010

Revision: Similarity Function

Contributing factors for paper recommendation:



Similarity is high when:

# of chains is high, chains are short, chains are heavy

Revision: Basic Assumptions

In theory:

Triangle inequality
Doubling dimension is $o(\log n)$

Typical web dataset has separation effect

For almost all i, j : $1/2 \leq d(p_i, p_j) \leq 1$

Classic methods fail:

Branch and bound algorithms visit every object
Doubling dimension is at least $\log n/2$

History

Navin Goyal, YL, Hinrich Schütze, WSDM 2008:

- Combinatorial framework: new approach to data mining problems that does not require triangle inequality
- Nearest neighbor algorithm

YL, Shengyu Zhang, SODA 2009:

- Better nearest neighbor search
- Detecting near-duplicates, navigability for small worlds

Dominique Tschopp, Suhas Diggavi, ArXiv 2009:

- LSH-like combinatorial algorithm

Combinatorial Framework vs. Ideal Algorithm

- − Any dataset
- + Any data model
- + Any dimension
- + Exact nearest neighbor
- + Provable efficiency
- +* Zero probability of error
- +* Near-linear space
- +* Logarithmic search time

Outline

1. Combinatorial Framework
2. Combinatorial Random Walk
3. Combinatorial Nets Algorithm
4. Applications of Combinatorial Framework

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Combinatorial Framework

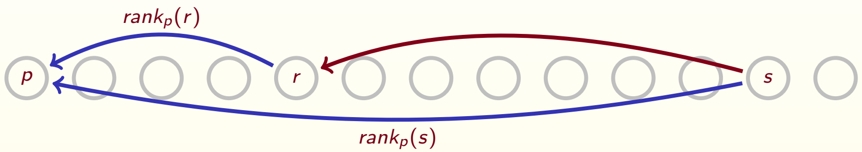
Comparison Oracle

- Dataset $p_1, \ldots, p_n$
- Objects and distance (or similarity) function are NOT given
- Instead, there is a comparison oracle answering queries of the form:

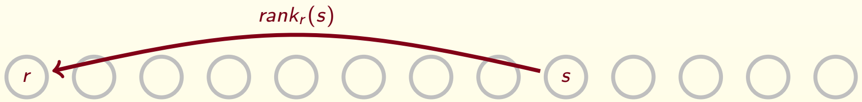
Who is closer to A: B or C?

Disorder Inequality

Sort all objects by their similarity to p:



Then by similarity to r:



Dataset has **disorder** D if

$\forall p, r, s: \quad rank_r(s) \leq D(rank_p(r) + rank_p(s))$

Combinatorial Framework

$=$

Comparison oracle

Who is closer to A: B or C?

$+$

Disorder inequality

$rank_r(s) \leq D(rank_p(r) + rank_p(s))$

Combinatorial Framework: FAQ

- Disorder of a metric space? Disorder of $\mathbb{R}^k$?
- In what cases disorder is relatively small?
- Experimental values of D for some practical datasets?

Disorder vs. Others

- If expansion rate is c, disorder constant is at most c^2
- Doubling dimension and disorder dimension are incomparable
- Disorder inequality implies combinatorial form of "doubling effect"

Combinatorial Framework: Pro & Contra

Advantages:

- Does not require triangle inequality for distances
- Applicable to any data model and any similarity function
- Require only comparative training information

Limitation: worst-case form of disorder inequality

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Combinatorial Random Walk

Ranwalk Informally

Hierarchical greedy navigation:

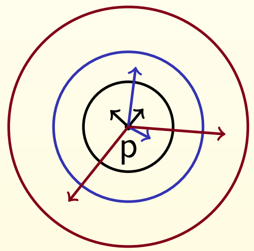
1. Start at random city p_1
2. Among all airlines choose the one going most closely to q, move there (say, to p_2)
3. Among all railway routes from p_2 choose the one going most closely to q, move there (p_3)
4. Among all bus routes from p_3 choose the one going most closely to q, move there (p_4)
5. Repeat this $\log n$ times and return the final city

Ranwalk: Data structure

Set $D' = 6D \log\log n$
For every object p in database S
choose at random:

- D' pointers to objects in $S = B(p, n)$
- D' pointers to objects in $B(p, \frac{n}{2})$

  . . .
- D' pointers to objects in $B(p, D')$

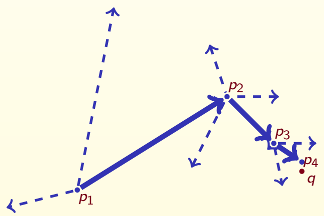


Ranwalk: Search via Greedy Walk

- Start at random point p_0
- Check endpoints of 1st level pointers, move to the best one p_1

  ...

- Check all D endpoints of bottom-level pointers and return the best one $p_{\log n}$



Analysis of Ranwalk

Assume that database points together with query point $S \cup \{q\}$ satisfy disorder inequality with constant D:

$$\mathrm{rank}_x(y) \leq D(\mathrm{rank}_z(x) + \mathrm{rank}_z(y)).$$

Then for any error probability δ Ranwalk will use the following resources:

- Preprocessing space: $\mathcal{O}(D \log n(\log\log n + \log 1/\delta)$
- Preprocessing time: $\mathcal{O}(n^2 \log n)$
- Search time $\mathcal{O}(D \log n(\log\log n + \log 1/\delta) + D^3)$

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Combinatorial Nets Algorithm

Navigating DAG

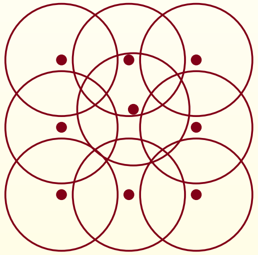
- $\log n$ layers
- $C_{i-1} \subset C_i$
- Down-degree is bounded ($poly(D)$)
- Search via "greedy dive"

Combinatorial Net

A subset $R \subseteq S$ is called a **combinatorial r-net** iff the following two properties holds:

Covering: $\forall y \in S, \exists x \in R,$ s.t. $\text{rank}_x(y) < r$.

Separation: $\forall x_i, x_j \in R,$ $\text{rank}_{x_i}(x_j) \geq r$ OR $\text{rank}_{x_j}(x_i) \geq r$



How to construct a combinatorial net?
What upper bound on its size can we guarantee?

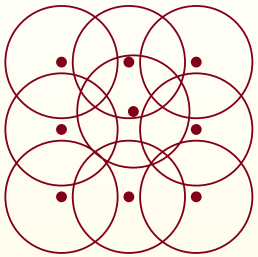
Basic Data Structure

Combinatorial nets:
For every $0 \leq i \leq \log n$, construct a $\frac{n}{2^i}$-net

Pointers, pointers, pointers:

- Direct & inverted indices: links between centers and members of their balls
- Cousin links: for every center keep pointers to close centers on the same level
- Navigation links: for every center keep pointers to close centers on the next level

Fast Net Construction



Theorem

Combinatorial nets can be constructed in $\mathcal{O}(D^7 n \log^2 n)$ time

Up'n'Down Trick

Assume your have $2r$-net for the dataset

To compute an r-ball around some object p:

1. Take a center p' of $2r$ ball that is covering p
2. Take all centers of $2r$-balls nearby p'
3. For all of them write down all members of theirs $2r$-balls
4. Sort all written objects with respect to p and keep r most similar ones.

Search by Combinatorial Nets

- $\log n$ layers
- $C_{i-1} \subset C_i$
- Down-degree is bounded ($poly(D)$)
- Search via "greedy dive"

Navigating DAG:

- Layer i: combinatorial net with radius $n/2^i$
- Down-links from p: members of next layer $i+1$ having rank to p at most $3D^2 \frac{n}{2^{i+1}}$

Analysis of Combinatorial Nets

Assume $S \cup \{q\}$ has disorder constant D

Theorem

There is a deterministic and exact algorithm for nearest neighbor search:

- *Preprocessing:* $\mathcal{O}(D^7 n \log^2 n)$
- *Search:* $\mathcal{O}(D^4 \log n)$

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Applications of Combinatorial Framework

Near-Duplicates

Assume, comparison oracle can also tell us whether $\sigma(x, y) > T$ for some similarity threshold T

Theorem

All pairs with over-T similarity can be found deterministically in time

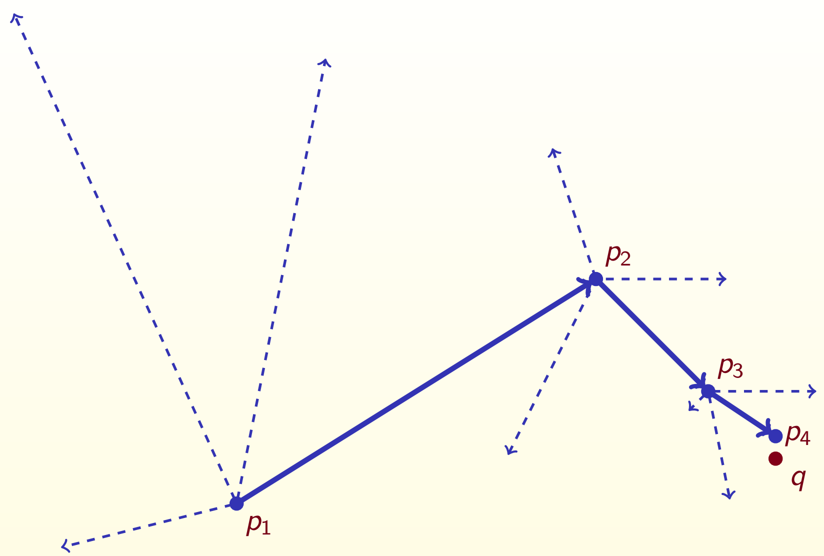
$$poly(D)(n \log^2 n + |Output|)$$

Visibility Graph

Theorem

For any dataset S with disorder D there exists a ***visibility graph****:*

- $poly(D) n \log^2 n$ *construction time*
- $\mathcal{O}(D^4 \log n)$ *out-degrees*
- *Naïve greedy routing*
  deterministically reaches
  exact nearest neighbor of the given target q
  in at most $\log n$ steps

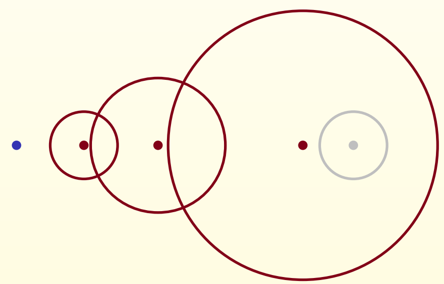


Definition of Visibility

A center c_i in the $\frac{n}{2^i}$-net is visible from some object p iff

$$rank_p(c_i) \leq 3D^2 \frac{n}{2^i}$$

Interpretation: the farther you are the larger radius you need to be visible



Directions for Further Research

Future of Combinatorial Framework

- What if disorder inequality has exceptions?
- Insertions, deletions, changing metric
- Experiments & implementation
- Metric transformations
- Unification challenge: disorder + doubling = ?

Summary

- Combinatorial framework:
  comparison oracle + disorder inequality
- New algorithms:
  Nearest neighbor search
  Deterministic detection of near-duplicates
  Navigability design

Thanks for your attention!
Questions?

Links

http://yury.name/esscass/

Yury Lifshits and Shengyu Zhang
Combinatorial Algorithms for Nearest Neighbors, Near-Duplicates and Small-World Design
http://yury.name/papers/lifshits2008similarity.pdf

Navin Goyal, Yury Lifshits, Hinrich Schütze
Disorder Inequality: A Combinatorial Approach to Nearest Neighbor Search
http://yury.name/papers/goyal2008disorder.pdf

Dominique Tschopp, Suhas Diggavi
Approximate nearest neighbor search through comparisons
http://arxiv.org/pdf/0909.2194