Obfuscation Around Point Functions

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Assumptions in Cryptography

Polynomial reduction:

If there exists an algorithm breaking my cryptosystem in polynomial time than it is also possible to solve some well-known hard problem in polynomial time

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Some classical assumptions:

- Factoring Integers is hard (no polynomial algorithm)
- Discrete Logarithm is hard
- One-Way Functions exist

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Property Hiding

Property hiding informally:

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$$\forall A : |Pr\{A(O(f)) = \pi(f)\} - 1/2| = \nu(|f|)$$

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Question: differences with black-box security?

Is There Hidden Functionality in the Program?

prog π_1^w;
var x:string, y:bit;
input(x);
if $x = w$ then y:=1 else
y:=0;
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end of prog;

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prog π_0;
var x:string, y:bit;
input(x);
y:=0; output(y);
end of prog;

Task: Make this families indistinguishable.

Some Theoretical Background

One-Way Permutation is bijection from the set of all binary strings of length k to itself which is easy to compute and difficult to inverse.

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Usual construction of hard-core predicate: choose r by random and take any one way permutation F than given a pair $(F(x), r)$ its difficult to uncover $x \cdot r$.

Obfuscation for Hidden Functionality

```
prog Π
var x: string, y:bit;
const u, v:string, σ:bit;
input(x);
if ONE_WAY(x)=v then
     if x · u = σ then y:=1 else y:=0;
else y:=0;
output(y);
end of prog;
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Multi-point functions with output

$$P_{A,B}(x) = B_i \text{ iff } x = A_i$$

Random Oracle Model

Random function $R : B^n \rightarrow B^m$ is just a random element from the set of all functions from B^n to B^m

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In Random Oracle Model all participants (obfuscator, obfuscated program and adversary) have oracle access to a random function

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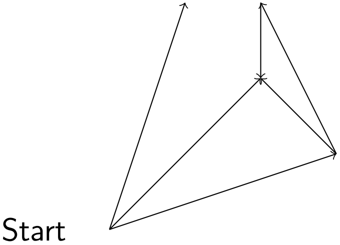
Which one?

Access Control Mechanism (1)

Informally:

- An unknown graph defining access to nodes
- Each edge has a password
- Start at start node
- Exponentially many access patterns



Access Control Mechanism (2)

- A directed multi-graph G on k vertices.
  $E = \{(u, v, i) : v = \mu_u^{(i)}\}$
- A set of passwords $\{\pi_e | e \in E\}$
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$$X_G((i_1, x_1), \ldots, (i_n, x_n)) = \begin{cases} v_n, \sigma_{v_n}, & \text{if } \exists v_0, \ldots, v_n \in [k] \text{ and} \\ & e_0, \ldots, e_{n-1} \in E \text{ such that} \\ & v_0 = 1, e_j = (v_j, v_{j+1}, i_j), \\ & \text{and } x_j = \pi_{e_j} \\ , & \text{otherwise} \end{cases}$$

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Claim: obfuscation of access functions can be reduced to that of multi-point functions

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New Assumption

There exists a polynomial-time computable permutation $\pi : B^n \longrightarrow B^n$ and a constant c such that for every polynomial $s(n)$ and every adversary A of size s for all sufficiently large n,

$$Pr[A(\pi(x)) = x] \leq s(n)^c/2^n$$

New Construction

Instead of $R(a)$ we will store:

$$h(x, \tau_1, \ldots, \tau_{3n}) = (\tau_1, \ldots, \tau_{3n}, \langle \pi(x), \tau_1 \rangle, \ldots, \langle \pi^{3n}(x), \tau_{3n} \rangle)$$

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Quiz in a Newspaper

- We publish some problem in a newspaper
- We want publish some additional check-yourself information
- We want that this information will contain almost zero information about the answer

Number-theoretic Construction

Let the answer is x
We will publish:

$$H(x) = (r^2, r^{2h(x)})$$

If hash function h is collision-free, then H is black-box secure about x.

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Assumption: Let $p = 2q - 1$ and $a, b, c \in_R Z_q^*$. Then distributions $\langle g^a, g^b, g^{ab} \rangle$ and $\langle g^a, g^b, g^c \rangle$ are computationally indistinguishable

Hash-based Construction

Let the answer is x
We will publish:

$$H(x) = (r, h(r, h(x)))$$

Home Problem 2 and 3

2. Prove that probability of changing functionality in obfuscating point functions by using random oracle from B^n to B^{3n} is negligible
3. Prove that $s^c(n)/2^n$, where s is a polynomial and c is a constant, is a negligible function

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Main points:

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- It is possible to obfuscate point functions with black-box security
- Security proofs are based on various cryptographic assumptions: existence of one-way functions, Diffie-Hellman indistinguishability, random oracle model
- To be honest, all solutions obsuscate data rather than algorithm

Reading List

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Thanks for attention. Questions?