# Processing Compressed Texts: A Tractability Border

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Combinatorial Pattern Matching 2007

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$=?$


Can we say whether they are equal without unpacking them?

# Outline

1. Popular topic in stringology: **algorithms for compressed texts**

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2. New algorithm for fully compressed pattern matching
3. Tractability border for processing compressed texts
4. Open problems

# Part I

## Formulating the problem

# Straight-line Programs: Definition

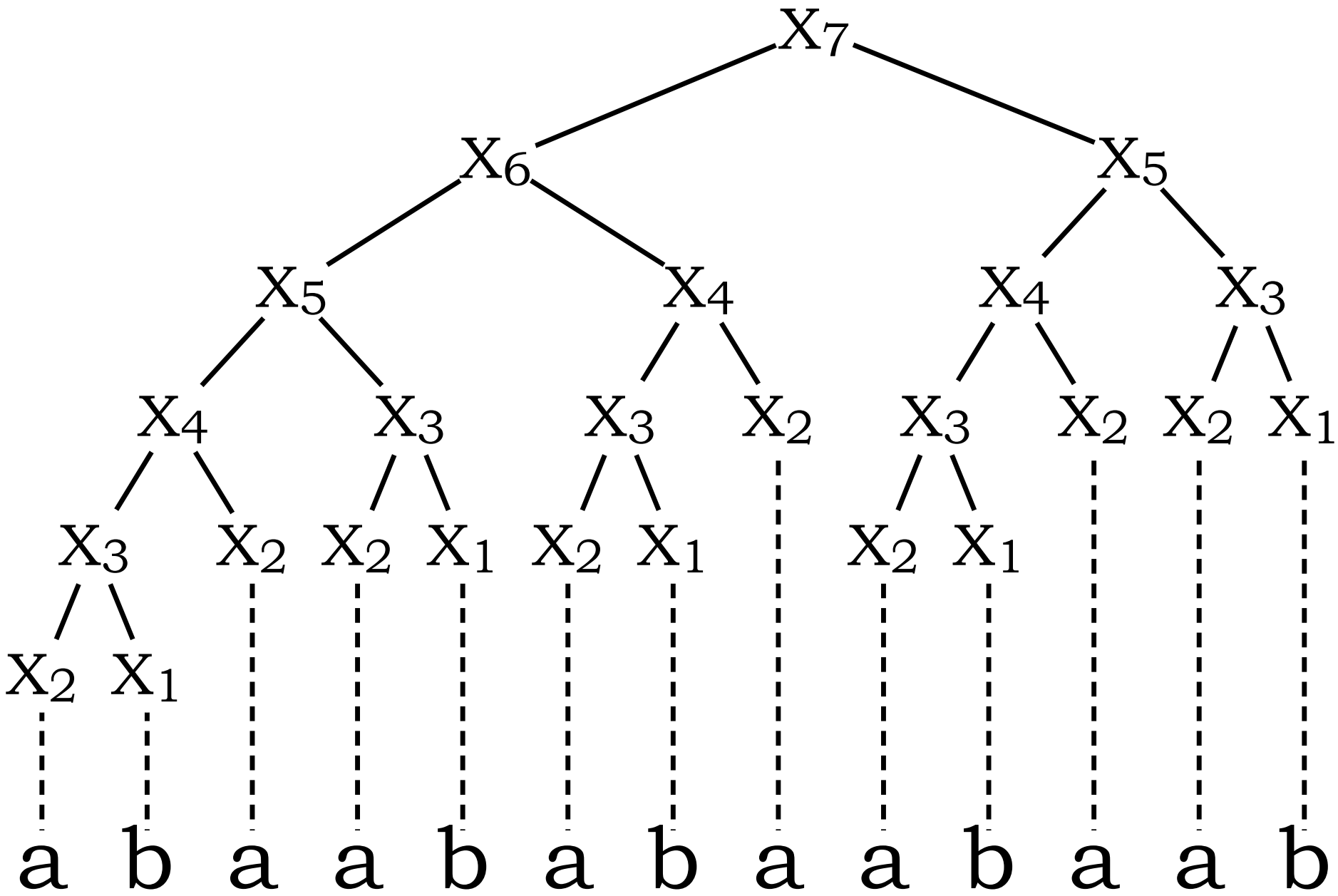
**Straight-line program** (SLP) is a context-free grammar generating **exactly one** string

Two types of productions:
$X_i \longrightarrow a$ and $X_i \longrightarrow X_p X_q$

**Example**

**abaababaabaab**

$$X_1 \longrightarrow b$$
$$X_2 \longrightarrow a$$
$$X_3 \longrightarrow X_2 X_1$$
$$X_4 \longrightarrow X_3 X_2$$
$$X_5 \longrightarrow X_4 X_3$$
$$X_6 \longrightarrow X_5 X_4$$
$$X_7 \longrightarrow X_6 X_5$$


X7
X6
X5
X5
X4
X4
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X3
X3
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a b a a b a b a a b a a b

# Rytter Theorem

## Theorem (Rytter'03)

*Resulting archive of most practical compression methods can be transformed into SLP generating the same original text. Two nice properties:*

1. *SLP has almost the same size as initial archive*
2. *Transformation is fast*

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**Notable exception:** compression schemes based on Burrows-Wheeler transform

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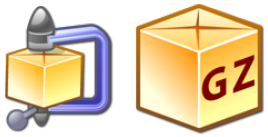
- **Connections to practical problems:**
  software verification, database compression, multimedia search
- **Complexity surprises:**
  many problems have unexpected computational complexity when input is given in compressed form

# Comparison Problems




Consider two straight-line programs $\mathcal{P}, \mathcal{Q}$ generating texts $P$ and $Q$. Tasks:

1. Determine whether $P = Q$
2. **Fully compressed pattern matching:** check whether $P$ is a substring of $Q$
3. Compute Hamming distance between $P$ and $Q$

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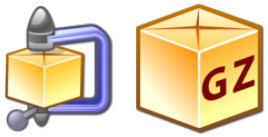


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Let $m, n$ be the sizes of $\mathcal{P}, \mathcal{Q}$. Best previously known result is $\mathcal{O}(n^2 m^2)$ algorithm for (1) and (2) **MST'97**

# Example

Do they generate the same text?

$$
\begin{aligned}
X_1 &\to b \\
X_2 &\to a \\
X_3 &\to X_2 X_1 \\
X_4 &\to X_3 X_2 \\
X_5 &\to X_4 X_3 \\
X_6 &\to X_5 X_4
\end{aligned}
\qquad
\begin{aligned}
Y_1 &\to b \\
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# Part II

## Fully compressed pattern matching

# New FCPM Algorithm

**MAIN RESULT 1**

Fully compressed pattern matching can be solved in $\mathcal{O}(n^2 m)$ time

# Dynamic Programming (1/2)

$$X_1 \to b$$
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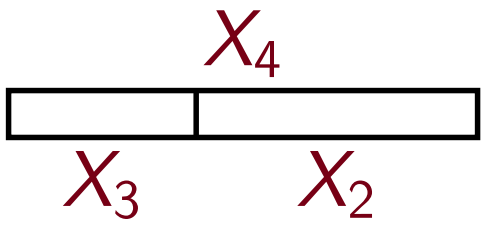
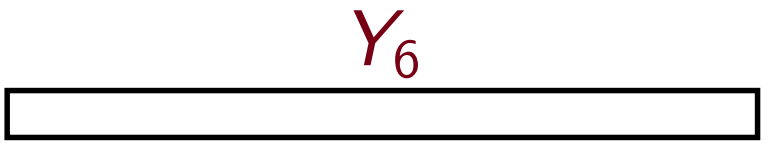
For every $i, j$ we compute occurrences of $X_i$ in $Y_j$ in lexicographic order of pairs: from $(1, 1)$ to $(6, 7)$

# Dynamic Programming (2/2)

Searching 

in 


From previous steps of dynamic programming:

- $X_3$ occurrences in $Y_6$
- $X_2$ occurrences in $Y_6$.

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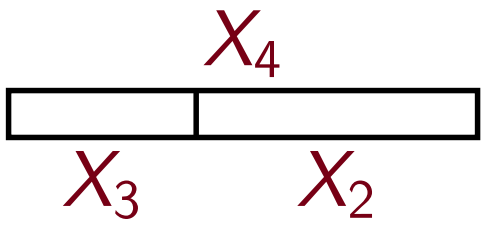
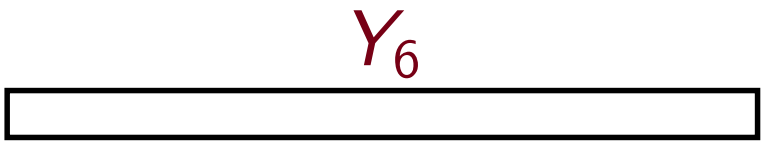
Searching 

in 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Take $X_3$ occurrences, $X_2$ occurrences, shift and intersect!

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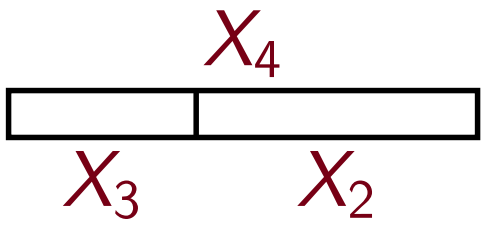
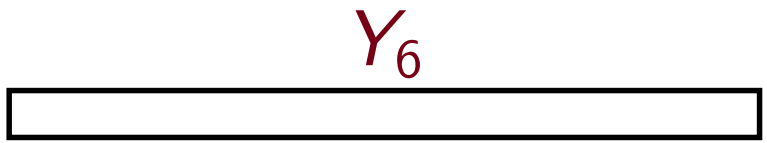
Searching 

in 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Complexity of one step of dynamic programming: $\mathcal{O}(n)$

Number of steps in dynamic programming: $\mathcal{O}(nm)$

# Part III

## Tractability Border

# Check Your Intuition

When two SLPs of sizes $n, m$ are given we can check equivalence between texts generated by them in time $\mathcal{O}(\min(n^2 m, m^2 n))$

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What is the complexity of computing Hamming distance between two SLP-generated texts?

**MAIN RESULT 2**

The problem of computing Hamming distance between SLP-generated texts is #P-complete

# Tractability Border

When input texts are compressed:

**$\exists$ poly algorithm:**

**GKPR'96** Equivalence
**GKPR'96** Regular Language Membership
**KPR'95** Fully Compressed Pattern Matching
**CGLM'06** Window Subsequence Matching
**GKPR'96** Shortest Period
**L'07** Shortest Cover

**At least NP-hard:**

**L'07** Hamming distance
**Lohrey'04** Context-Free Language Membership
**LL'06** Fully Compressed Subsequence Matching
**BKLPR'02** Two-dimensional Compressed Pattern Matching

# Part IV

# Open Problems

# OP1: Longest Common Substring

**Input:** Two SLPs generating texts $P$ and $Q$

**Task:** Compute the length of the longest common substring of $P$ and $Q$

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Can we do it in polynomial time?

# OP2: SLP for OR

**Input:** Two SLPs of sizes $n, m$ generating binary strings $P$ and $Q$ of the same length

**Task:** Compute a close-to-minimal SLP generating "bitwise OR between $P$ and $Q$"

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**Task:** Compute a close-to-minimal SLP generating "bitwise OR between $P$ and $Q$"

Can we do it in time $poly(n + m + output)$?

# OP3: Edit Distance

**Input:** Text $P$ of length $l$ and SLP of size $m$ generating text $Q$

**Input:** Compute edit distance between $P$ and $Q$

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**Input:** Text $P$ of length $l$ and SLP of size $m$ generating text $Q$

**Input:** Compute edit distance between $P$ and $Q$

Can we do it in $\mathcal{O}(lm)$ time?

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- Straight-line programs is a wide-accepted model of compressed text
- Fully compressed pattern matching can be solved in $\mathcal{O}(n^2 m)$ time
- Computing Hamming distance between compressed texts is #P-complete
- Open problems: longest common substring, bitwise OR and edit distance for compressed texts

# Last Slide

Search "**Lifshits**" or visit http://logic.pdmi.ras.ru/~yura/

Yury Lifshits

Processing Compressed Texts: A Tractability Border

*CPM'07* // on-line version

Yury Lifshits and Markus Lohrey

Querying and Embedding Compressed Texts

*MFCS'06* // on-line version

Patrick Cégielski, Irène Guessarian, Yury Lifshits and Yuri Matiyasevich

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# Thank you for your attention! Questions?