Similarity Search: a Web Perspective

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Google Tech Talk

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Similarity Search in a Nutshell

Input: Set of objects

Task: Preprocess it





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Roadmap

1 Similarity Search in Web

2 Similarity Search in Theory

3 Revising the Problem

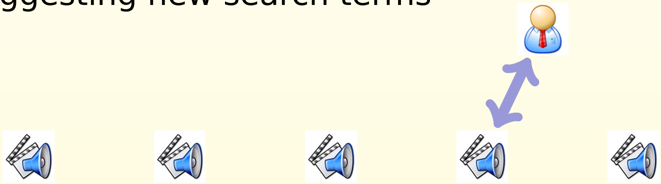
4 New Algorithms

1

Similarity Search in Web

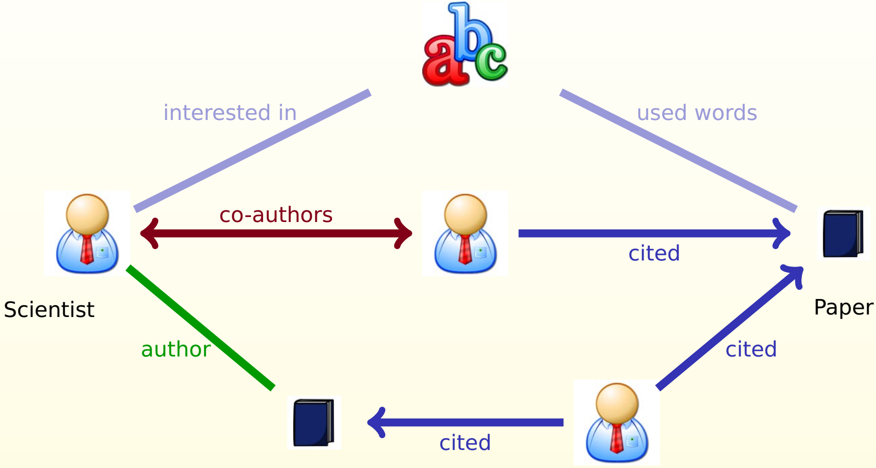
Similarity Search vs. Web

- Recommendations (movies, books...)
- Personalized news aggregation
- Ad targeting
- "Best match" search
  Resume, job, BF/GF, car, apartment
- Co-occurrence similarity
  Suggesting new search terms



Similarity in Networks

Similarity chart for paper recommendation:



Similarity is high when:

# of chains is high, chains are short, chains are heavy

2

Similarity Search in Theory

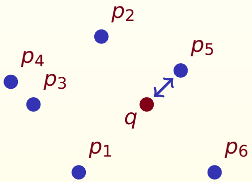
Nearest Neighbor Search

Search space: object domain $\mathbb{U}$, distance function d

Input: database $S = \{p_1, \ldots, p_n\} \subseteq \mathbb{U}$

Query: $q \in \mathbb{U}$

Task: find $\operatorname{argmin}_{p_i} d(p_i, q)$



Data Models:

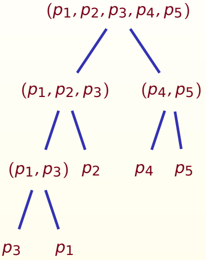
- General metric space:
  triangle inequality + oracle access
- k-dimensional Euclidean space with Euclidean, Manhattan, L_p or angle metric
- Strings with Hamming or Levenshtein distance
- Finite sets with Jaccard metric $d(A, B) = 1 - \frac{|A \cap B|}{|A \cup B|}$

Which One to Use?

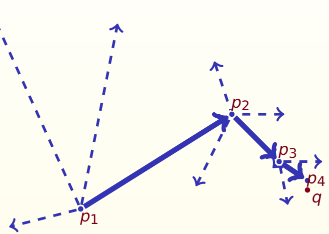
Sphere Rectangle Tree Orchard's Algorithm k-d-B tree Geometric near-neighbor access tree Excluded middle vantage point forest mvp-tree Fixed-height fixed-queries tree AESA Vantage-point tree LAESA R*-tree Burkhard-Keller tree BBD tree Navigating Nets Voronoi tree Balanced aspect ratio tree Metric tree vp^s-tree M-tree Locality-Sensitive Hashing SS-tree R-tree Spatial approximation tree Multi-vantage point tree Bisector tree mb-tree Cover tree Hybrid tree Generalized hyperplane tree Slim tree Spill Tree Fixed queries tree X-tree k-d tree Balltree Quadtree Octree Post-office tree

Four Famous Techniques

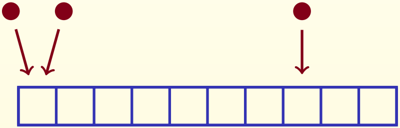
Branch and bound



Greedy walks

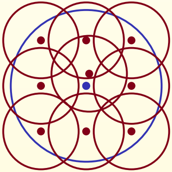


Mappings: LSH, random projections, minhashing



Epsilon nets

Works for small intrinsic dimension



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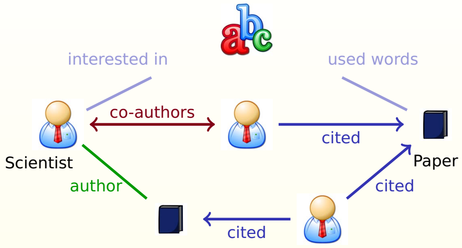
Nearest Neighbors: Revising the Problem

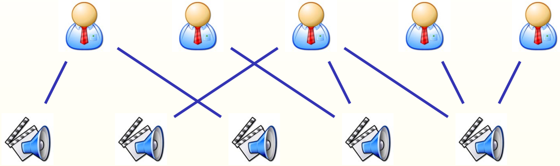
Revision: Data Model



- Several types of nodes and (weighted) edges, restrictions on degrees
- Similarity chart: List of “contributing chains”
- Similarity (relevance): sum of weight products over all contributing chains

Similarity Search in Bipartite Graphs

n vertices
degree $\leq k$



m vertices

Dataset: bipartite graph

Person-person similarity: # of 2-step chains

Person-movie similarity: # of 3-step chains

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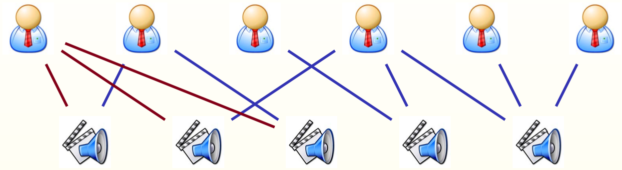

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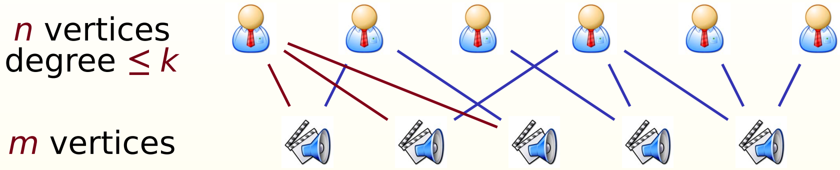

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Open problem:

Existence of similarity search with $poly(m, n)$ preprocessing and $poly(k, \log n, \log m)$ query time

Revision: Basic Assumptions

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Typical **web dataset** has separation effect

$$\text{For almost all } i, j: \quad 1/2 \le d(p_i, p_j) \le 1$$

Example: Jackard metric for # of joint friends

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Example: Jackard metric for # of joint friends

Corollaries:

In general metric space exact problem is intractable

Branch and bound algorithms visit every object

Doubling dimension is at least $\log n/2$

Revision: Notion of Success

In theory:

c-approximate algorithm returns $p: \quad d(p, q) \leq c \cdot d(p_{NN}, q)$

Polynomial preprocessing & sublinear search algorithm [AI06]

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Open problem:

Existence of polynomial preprocessing & sublinear search approximate algorithm for Euclidian space with cosine similarity

Revision: Dynamic Aspects

In theory:

Handling insertions & deletions

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In theory:

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Web:

Adding & removing edges

Affects many pairwise similarities

Weights are changing

Example: # of votes/comments on `Digg.com`

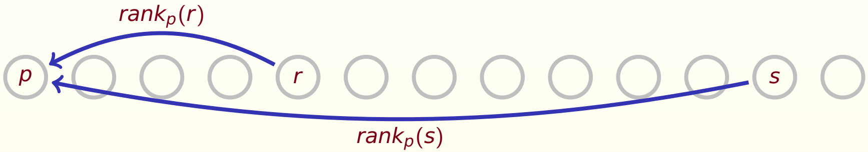
General formula for similarity is changing

4

New Algorithms for Similarity Search

Concept of Disorder

Sort all objects by their similarity to p:



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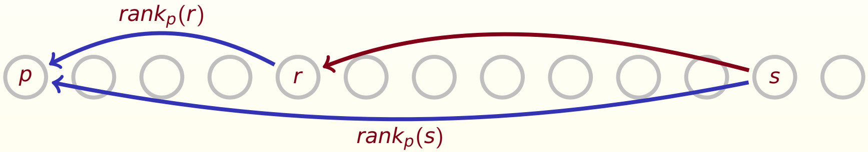
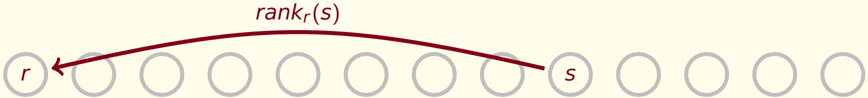


Then by similarity to r:



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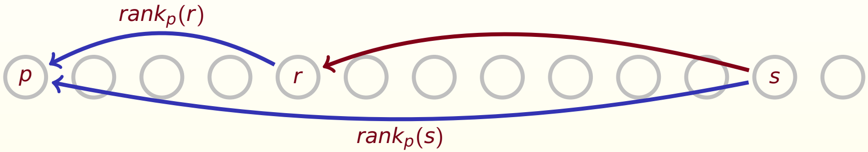

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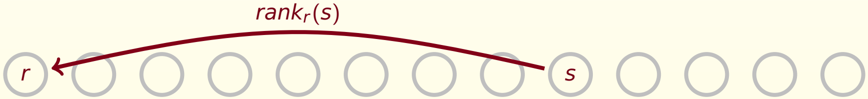
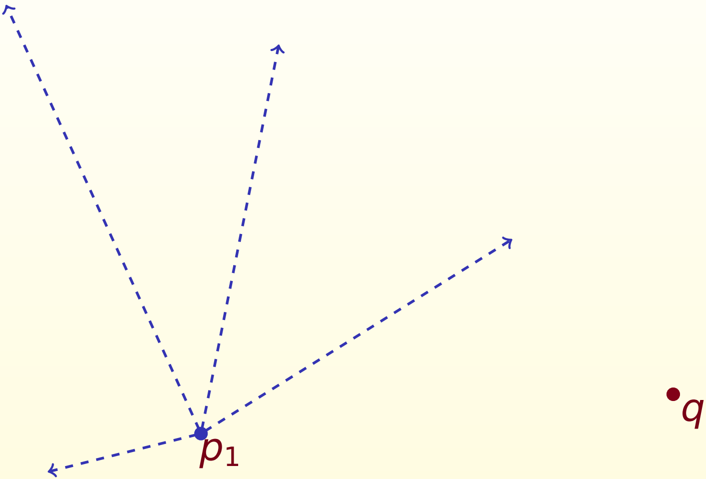


Dataset has **disorder** D if

$$\forall p, r, s: \quad rank_r(s) \leq D(rank_p(r) + rank_p(s))$$

Ranwalk Algorithm [GLS08]

Similarity search with roughly $\mathcal{O}(Dn \log n)$ data structure and $\mathcal{O}(D \log n)$ search time



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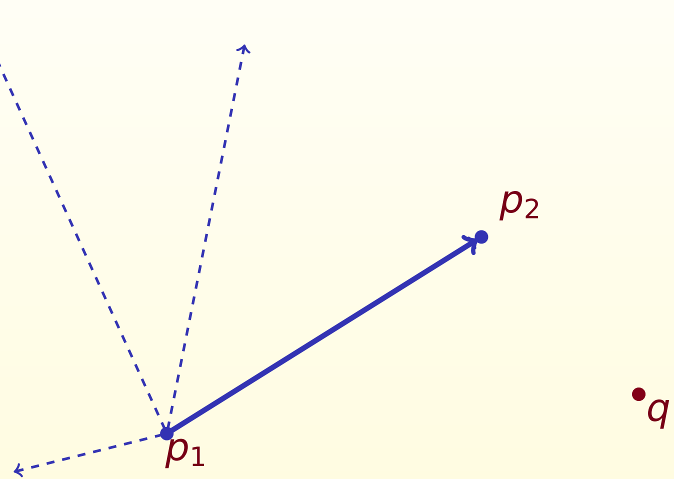

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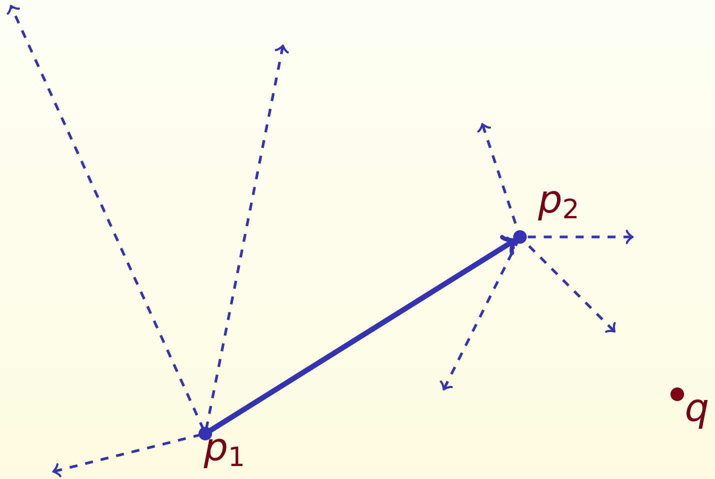

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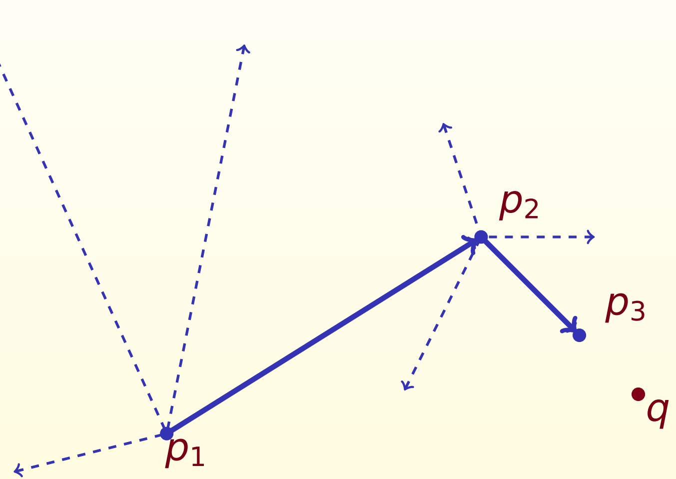

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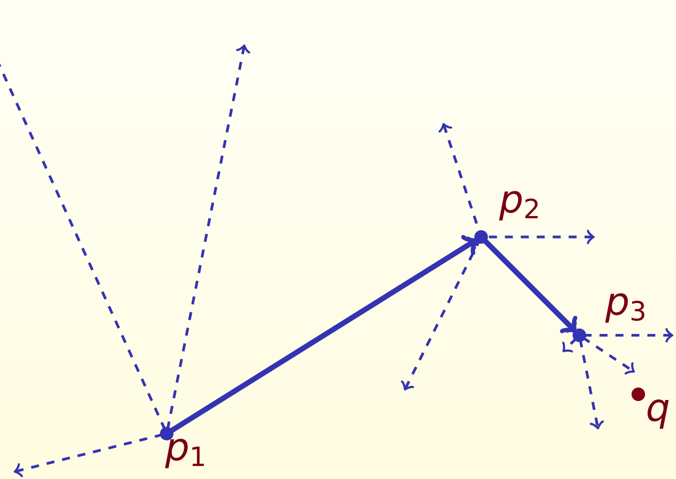

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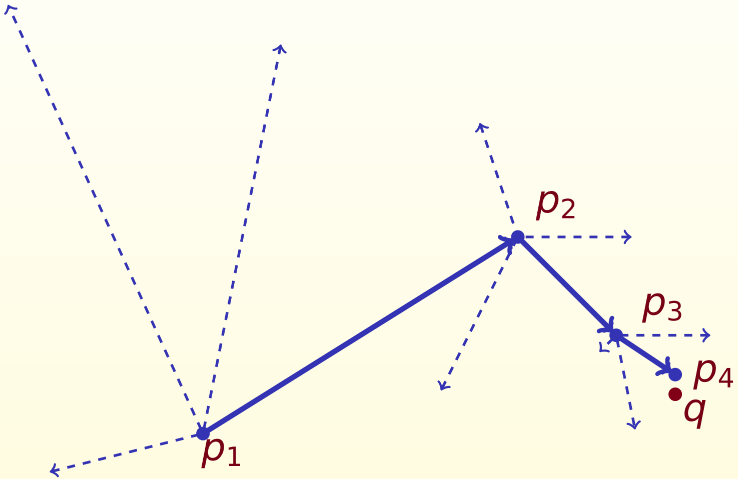
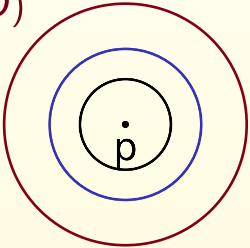


Ranwalk: Data structure

Set $D' = 6D \log\log n$
For every object p in database S
choose at random:

- D' pointers to objects in $S = B(p, n)$
- D' pointers to objects in $B(p, \frac{n}{2})$

  ...
- D' pointers to objects in $B(p, D)$



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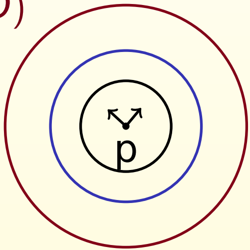

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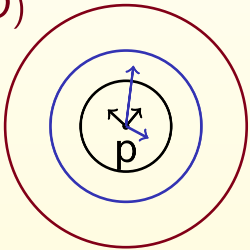

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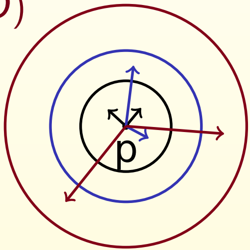
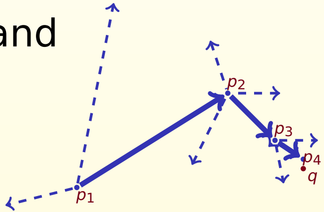


Ranwalk: Search via Greedy Walk

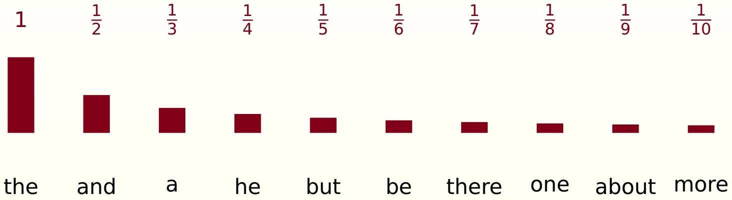
- Start at random point p_0
- Check endpoints of 1st level pointers, move to the best one p_1

  . . .

- Check all D endpoints of bottom-level pointers and return the best one $p_{\log n}$



Zipf Model



- Terms $t_1, \ldots, t_m$
- To generate a document we take every t_i with probability $\frac{1}{i}$
- Database is n independently chosen documents
- Similarity between documents is defined as the number of common terms

Magic Level Theorem [HLN07]

For **magic level** $q = \sqrt{2 \log_e n}$:

1. Any match: W.h.p. the best document in database has $q \pm \varepsilon$ overlap with query document

2. Prefix match: W.h.p. there is a document in database containing $q \pm \varepsilon$ of top frequent terms of query document

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Best prefix match is much easier to search for!

Questions to Google

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- **Announce the winner:** Which similarity search algorithms do you use?
- **Google datasets:** Give us benchmarks in ad targeting, news aggregation, citation networks

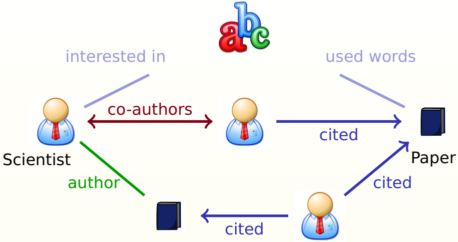
Sponsored Links

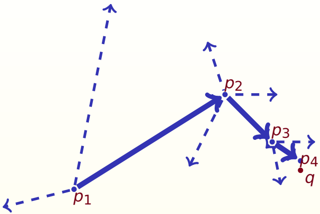
- http://yury.name

- Yury Lifshits
  Nearest Neighbors and Similarity Search
  Tutorial, bibliography, people, links, open problems
  http://simsearch.yury.name

- Navin Goyal, Yury Lifshits, Hinrich Schütze
  Disorder Inequality: A Combinatorial Approach to Nearest Neighbor Search
  http://yury.name/papers/goyal2008disorder.pdf

- Benjamin Hoffmann, Yury Lifshits, Dirk Novotka
  Maximal Intersection Queries in Randomized Graph Models
  http://yury.name/papers/hoffmann2007maximal.pdf





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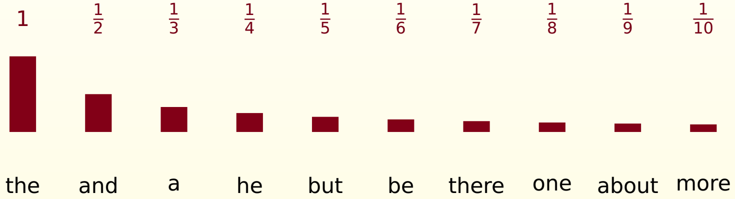

Thanks for your attention! Questions?