

Algorithms for Nearest Neighbors

Background and Two Challenges

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Outline

1 Formulating the Problem

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- 2 Nearest Neighbors for Texts

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- 3 Proving Hardness of Nearest Neighbors

Part I

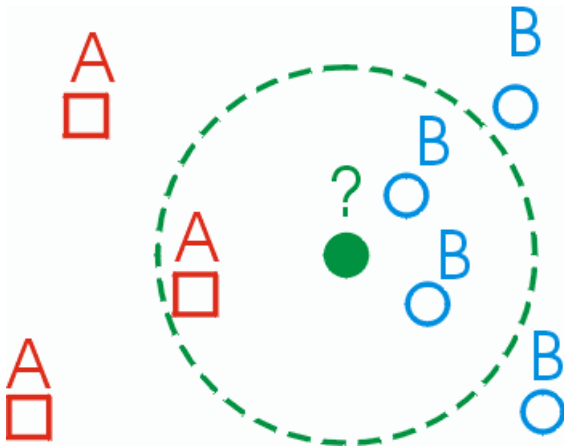
Formulating the Problem

Informal Problem Statement

To preprocess a database of n objects so that given a query object, one can effectively determine its nearest neighbors in database

First Application (1960s)

Nearest neighbors for classification:



Picture from <http://cgm.cs.mcgill.ca/~soss/cs644/projects/perrier/Image25.gif>

Applications

What applications of nearest neighbors do you know?

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- Text classification
- Statistical data analysis, e.g. medicine diagnosis
- Pattern recognition: characters, faces
- Code plagiarism detection
- Coding theory
- Data compression

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- **Web:** recommendation systems, on-line ads, personalized news aggregation, long queries in web search, near-duplicates detection

Data Model in General

Formalization for nearest neighbors consists of:

- Representation format for objects
- Similarity function

Basic Data Models (1/2)

- Vector Model
 - Similarity: l^2 , scalar product, cosine

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- String Model
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- Black-box model
 - Similarity: given by oracle
 - The only knowledge is triangle inequality

Basic Data Models (2/2)

- Set Model
 - Similarity: size of intersection

Basic Data Models (2/2)

- Set Model
 - Similarity: size of intersection
- Small graphs
 - Similarity: structure/labels matching

Algorithmic Approaches to NN

- Divide and conquer
- Traversal techniques
- Look-up techniques
- Contractive and low-distortion embeddings
- Tournament algorithms

Part II

Nearest Neighbors for Texts

Sparse Vector Model

Database: points in R^d ,
every point has at most $k \ll d$ nonzero coordinates

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$\text{poly}(n + d)$ for preprocessing time,
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Open Problem: solve NN for sparse vector model
within given constraints

Inverted Index

Preprocessing:

For every term store a list of all documents in database with nonzero weight on it

Query processing:

Retrieve all points that have at least one common term with the query document;
Perform linear scan on them

Rare-Point Method

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Probabilistic Analysis in a Nutshell

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- We construct a solution that is efficient/accurate with high probability over “random” input/query

Zipf Model

- Terms t_1, \dots, t_m
- To generate a document we take every t_i with probability $\frac{1}{i}$
- Database is n independently chosen documents
- Query document has exactly one term in every interval $[e^i, e^{i+1}]$
- Similarity between documents is defined as the number of common terms

Magic Level Theorem

Magic Level $q = \sqrt{2 \log_e n}$

Theorem (Hoffmann, Lifshits and Nowotka, CSR'07)

- 1 *With very high probability there exists a document in database having $q - \epsilon$ **top** terms of query document*
- 2 *With very small probability there exists a document in database having **any** $q + \epsilon$ overlap with query document*

Part III

Proving Hardness of Nearest Neighbors

Inclusions with Preprocessing (1/2)

Input

Family \mathcal{F} of subsets of U

Query task

Given a set $f_{new} \subseteq U$ to decide
whether $\exists f \in \mathcal{F} : f_{new} \subseteq f$

Constraints

Data storage after preprocessing $poly(|\mathcal{F}| + |U|)$

Time for query processing $poly(|U|)$

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Open problem: is there an algorithm satisfying given constraints?

Inclusions with Preprocessing (2/2)

Reformulation in SAT style:

Input

DNF formula \mathcal{F} on n variables, without negations

Query task

Given an assignment x to evaluate $\mathcal{F}(x)$

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Data storage after preprocessing $poly(|\mathcal{F}|)$

Time for query processing $poly(n)$

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“NP Analogue” for Search Problems

Every problem in **SEARCH class** is characterized by poly-time computable Turing Machine M :

Input

Strings x_1, \dots, x_n , $|x_i| = m$

Query task

Given string y of length m to answer whether $\exists i : M(x_i, y) = \text{yes}$

Tractable problems in SEARCH

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Tractable solution

Preprocessing in $\text{poly}(m, n)$ space

Query processing in $\text{poly}(m, \log n)$ time with RAM access to preprocessed database

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Inclusions is in SEARCH. Is it tractable?

Complete problems in SEARCH (1/2)

Program Search problem:

Input

Turing machines P_1, \dots, P_n

Query task

Given string y of length m to answer whether $\exists i : P_i(y) = \text{yes}$ after at most m steps

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Given string y of length m to answer whether $\exists i : P_i(y) = \text{yes}$ after at most m steps

Open problem: is Program Search tractable?

Complete problems in SEARCH (2/2)

Parallel Run problem:

Input

$x_1 \dots, x_n$

Query task

Given poly-time computable P to answer whether $\exists i : P(x_i) = \text{yes}$

Complete problems in SEARCH (2/2)

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Given poly-time computable P to answer whether $\exists i : P(x_i) = \text{yes}$

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Conclusions

Call for Feedback

- Any relevant work?
- How to improve this talk for the next time?

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- **Give my open problems to your friends!**

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Thanks for your attention! Questions?

References (1/2)

Search “**Lifshits**” or visit <http://logic.pdmi.ras.ru/~yura/>



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<http://logic.pdmi.ras.ru/~yura/en/maxint-draft.pdf>



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Data structures and algorithms for nearest neighbor search in general metric spaces

<http://www.pnylab.com/pny/papers/vptree/vptree.ps>



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Inverted files for text search engines

<http://www.cs.mu.oz.au/~alistair/abstracts/zm06compsurv.html>



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Links to nearest neighbors implementations

<http://people.revoledu.com/kardi/tutorial/KNN/resources.html>

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<http://www.ece.tuc.gr/~vsam/csalgo/kleinberg-stoc97-nn.ps>



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Approximate nearest neighbors: towards removing the curse of dimensionality

<http://theory.csail.mit.edu/~indyk/nndraft.ps>



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P. Indyk

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