Querying and Embedding Compressed Texts

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Subsequence Matching (Embedding)

INPUT: pattern **TEAM** and text

I	N	T	E	R	N	A	T	I	O	N	A	L		S	Y	M	P	O	S	I	U	M		M	F	C	S

TASK: to check whether the text contains the pattern as a **subsequence** (i.e. gaps are allowed)

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Problem for this talk:

Given a COMPRESSED text and a COMPRESSED pattern can we solve embedding faster than just "unpack-and-search"?

Outline of the Talk

1. New topic in computer science: **algorithms for compressed texts**
2. Our problems and our results
3. Some proof ideas

Part I

What are **compressed** texts?

Can we do something interesting **without unpacking**?

Straight-line Programs: Definition

Straight-line program (SLP) is a

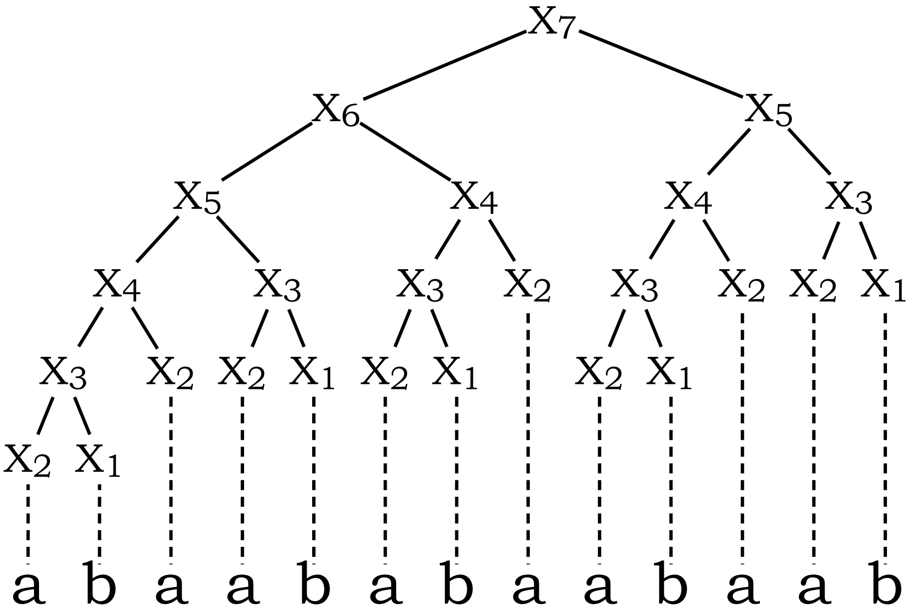
Context-free grammar generating **exactly one** string

Two types of productions: $X_i \longrightarrow a$ and $X_i \longrightarrow X_p X_q$

Example

abaababaabaab

$X_1 \longrightarrow b$
$X_2 \longrightarrow a$
$X_3 \longrightarrow X_2 X_1$
$X_4 \longrightarrow X_3 X_2$
$X_5 \longrightarrow X_4 X_3$
$X_6 \longrightarrow X_5 X_4$
$X_7 \longrightarrow X_6 X_5$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SLP = Compressed Text

Rytter, 2003: Consider the archive of size z obtained by LZ78,LZW or some dictionary-based compression method. Then we can in time $O(z)$ convert it to SLP of size $O(z)$ generating the same text.

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In the following by compressed text we mean an **SLP generating it**

Why algorithms on compressed texts?

Answer for algorithms people:

- Might be faster than "unpack-and-search"
- Saving storing space and transmitting costs
- Many fields with highly compressible data: statistics (internet log files), automatically generated texts, message sequence charts for parallel programs

Answer for complexity people:

- Some problems are hard in worst case. But they might be easy for **compressible** inputs
- New complexity relations. Similar problems have different complexities on compressed inputs

Problems on SLP-generated texts

∃ poly algorithms:

GKPR'96 Equivalence
GKPR'96 Regular Language Membership
GKPR'96 Shortest Period
L'06 Shortest Cover
L'06 Fingerprint Table
GKPR'96 Fully Compressed Pattern Matching
CGLM'06 Window Subsequence Matching

At least NP-hard:

L'06 Hamming distance
Lohrey'04 Context-Free Language Membership
BKLPR'02 Two-dimensional Compressed Pattern Matching

Part II

What are embedding and querying problems on compressed texts?

How computationally hard are they?

Querying and Embedding Compressed Texts

Compressed Embedding Problem:
INPUT: Two SLPs generating strings T and P
OUTPUT: YES if T contains P as a subsequence, otherwise NO

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Compressed Querying Problem:
INPUT: A SLP generating string T, position i, character a
OUTPUT: YES if $T_i = a$, otherwise NO

Compressed Embedding is Hard

GKPR'96 proved that **string matching** when both the text and the pattern are compressed has a polynomial algorithm.

Natural question: then what about **subsequence matching**?

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MAIN RESULT 1:

Compressed Embedding problem is NP-hard
Compressed Embedding problem is co-NP-hard.

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Compressed Embedding problem is co-NP-hard.
Compressed Embedding problem is Θ_2-hard

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The most used operation on compressed texts is decompressing.

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MAIN RESULT 2:

Compressed Querying problem is P-complete.

Part III

How to prove NP-hardness of Embedding?

How to prove co-NP-hardness of Embedding?

Proving NP-hardness

Classical reduction:

1. Take an NP-complete problem (**Subset Sum**)
2. For every instance of Subset Sum construct two straight line programs such that

$$\text{Embedding holds} \Leftrightarrow \text{Subset Sum has answer "Yes"}$$

Proving co-NP-hardness

Lemma (Yes-No symmetry):

For every SLPs X and Y we can in polynomial time construct SLPs X' and Y' such that:

$$\text{Embedding holds for } X \text{ and } Y$$
$$\Leftrightarrow$$
$$\text{Embedding does not hold for } X' \text{ and } Y'$$

Proving co-NP-hardness

Lemma (Yes-No symmetry):
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Corollary: NP-hardness implies co-NP-hardness

Summary

Main points:

- Compressed text = text generated by SLP
- For compressed texts querying is P-complete, embedding is Θ_2-hard
- Method: reduction from subset sum problem, “yes-no” symmetry

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Open Problems:

- What is exact complexity of Compressed Embedding problem (we know that it is somewhere between Θ_2 and PSPACE)?
- To construct $O(nm)$ algorithms for edit distance, where n is the length of T_1 and m is the **compressed size** of T_2

Last Slide

Yury Lifshits http://logic.pdmi.ras.ru/~yura/

Our relevant papers:

- Yury Lifshits and Markus Lohrey
  Querying and Embedding Compressed Texts
  MFCS'06.
- Yury Lifshits
  Solving Classical String Problems on Compressed Texts
  preprint at Arxiv:cs.DS/0604058, 2006.
- P. Cégielski, I. Guessarian, Yu. Lifshits and Yu. Matiyasevich
  Window Subsequence Problems for Compressed Texts
  CSR'06.
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  Word Problems and Membership Problems on Compressed Words
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Thanks for attention!