

Open Problems TO GO

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Open Problems TO GO:

- Short mathematical statement
- No background required
- Motivation (importance) is guaranteed

Today:

Three classic problems
Three problems from YL

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Classic Problems

Mean payoff games

Semi-Thue systems

Ulam conjecture (graph reconstruction)

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1.1. Rules of mean payoff games

Input for a **mean payoff game**:

- Weighted directed graph (integer weights)
- Graph does not contain simple cycles with zero sum
- Vertices are divided into disjoint sets A and B
- The starting vertex

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Rules of Mean Payoff Games

- Two players: Alice and Bob
- Players move the token over arcs
- Game starts from the starting vertex and it is infinite
- Alice plays from vertices of A , Bob from these of B
- Alice wins if the sum of already passed arcs goes to $+infty$
- Bob wins if the sum of already passed arcs goes to $-infty$

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Computational Problem

Given a game graph with an A, B decomposition and a starting vertex to determine the winner (and find the winning strategy)

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MPG is Very Challenging




MPG Problem belongs to $NP \cap co-NP$
Direct applications in μ -calculus verification

Known algorithms:

- Randomized algorithm $\mathcal{O}^*(2^{\sqrt{n}})$ expected time
- Deterministic algorithm $\mathcal{O}^*(2^n)$ time

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References

-  Y. Lifshits, D. Pavlov
Potential Theory for Mean Payoff Games
Journal of Mathematical Sciences, 2007
<http://yury.name/papers/lifshits2006fast.pdf>
-  M. Jurdziński, M. Paterson, U. Zwick
A deterministic subexponential algorithm for solving parity games
SODA'06
<http://www.dcs.warwick.ac.uk/~mju/Papers/JPZ07-manuscript.pdf>
-  H. Björklund, S. Vorobyov
A combinatorial strongly subexponential strategy improvement algorithm for mean payoff games
Discrete Applied Mathematics, 2007
<http://portal.acm.org/citation.cfm?id=1222484>

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Ulam Conjecture

A vertex-deleted subgraph of a graph G is a subgraph $G - v$ obtained by deleting a vertex v and its incident edges. The deck of a graph G is the family of (unlabelled) vertex-deleted subgraphs of G ; these are the cards of the deck. A reconstruction of a graph G is a graph H with the same deck as G . A graph G is reconstructible if every reconstruction of G is isomorphic to G .

Conjecture: every graph with at least three vertices is reconstructible

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Reference



J.A. Bondy

A graph reconstructor's manual

Surveys in Combinatorics, 1991

<http://www.ecp6.jussieu.fr/pageperso/bondy/research/papers/recon.ps>

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Semi-Thue Systems

Rewriting (α, β) rule allows to rewrite any $u\alpha v$ in $u\beta v$

Word problem: Given system of rules and two words w_1 and w_2 to decide whether one can be obtained from another by a sequence of such rules?

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Challenge

There is a system with three rules such that word problem is undecidable

Is word problem decidable or not for systems of one (two) rules?

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Reference

-  Y. Matiyasevich and G. Senizerguez
Decision Problems for Semi-Thue Systems with a Few Rules
LICS'96
<http://dept-info.labri.u-bordeaux.fr/~ges/termination.ps>

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Open Problems from YL

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Compressed Arithmetic




Input: Two grammars of size n, m generating binary strings P and Q of the same length

Task: Compute a close-to-minimal grammar generating “bitwise OR between P and Q ”

Can we do it in time $\text{poly}(n + m + \text{output})$?

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References

-  Yury Lifshits
Processing Compressed Texts: A Tractability Border
CPM'07
<http://yury.name/papers/lifshits2007processing.pdf>
-  Yury Lifshits and Markus Lohrey
Querying and Embedding Compressed Texts
MFCS'06
<http://yury.name/papers/lifshits2006querying.pdf>
-  Patrick Cégielski, Irène Guessarian, Yury Lifshits and Yuri Matiyasevich
Window Subsequence Problems for Compressed Texts
CSR'06
<http://yury.name/papers/cegielski2006window.pdf>

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Impossibility of Preprocessing

Input

Circuits C_1, \dots, C_n
of size $\text{poly}(m)$ with input size m

Query task

Given string y of length m to answer
whether $\exists i : C_i(y) = \text{yes}$

Constraints:

$\text{poly}(n, m)$ preprocessing
 $\text{poly}(\log n, m)$ search

Open problem: Is there a solution within
given constraints?

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Dual Problem

Input

Strings x_1, \dots, x_n of length m ,

Query task

Given circuit C of size $\text{poly}m$ with input
length m to answer whether $\exists i : C(x_i) = \text{yes}$

Constraints:

$\text{poly}(n, m)$ preprocessing
 $\text{poly}(\log n, m)$ search

Open problem: Is there a solution within
given constraints?

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Reference



Yury Lifshits

Algorithms for Nearest Neighbors: Classic Ideas, New Ideas

Talk at University of Toronto

[MP3 recording](#)

<http://yury.name/talks/toronto-talk.pdf>

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Positive Subgraph

Input

$n \times n$ bipartite graph (pretty sparse)
Weights on edges

Task

Find a $k \times k$ subgraph
with maximal average edge weight

Polynomial approximate algorithm?

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Reference



Y. Lifshits and D. Nowotka

Estimation of the click volume by large scale regression analysis

CSR'07

<http://yury.name/papers/lifshits2007click.pdf>



<http://www.netflixprize.com>

Voting

Which problem you like the most?

- Mean Payoff Games
- Ulam Conjecture
- Semi-Thue Systems
- Compressed Arithmetics
- Impossibility of Preprocessing
- Positive Subgraph

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Thanks for your attention!
Questions?