New Algorithms on Compressed Texts

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Tallinn
20/03/2006

FCPM: Problem Description

Fully Compressed Pattern Matching (FCPM)

INPUT: Compressed strings P and T
OUTPUT: Yes/No (whether P is a substring in T?)

Example

Text: abaa**baba**abaab
Pattern: **baba**

We know only compressed representation of P and T

Outline of the Talk

1. Processing Compressed Texts: Bird's Eye View
2. Fully Compressed Pattern Matching: Idea of a New Algorithm
   - Idea of a new algorithm
   - ★ Detailed description
3. More Algorithms and Some Negative Results
4. Conclusions and Open Problems

Processing Compressed Text

Central idea

If some text is highly compressible,
then it contains long identical segments
and therefore it is likely that we can solve
some problems more efficiently than in general case

Motivation

Reasons for algorithms on compressed texts:

- Potentially faster than "unpack-and-solve"
- Lower memory requirements
- Theoretical applications: word equations in PSPACE, pattern matching in message sequence charts

Real data with high level of repetitions:

- Genomes
- Internet logs, any statistical data
- Automatically generated texts

Straight-Line Programs

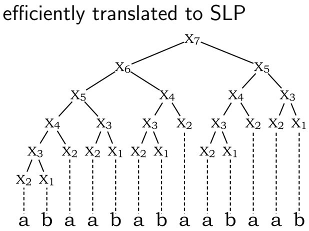
Straight-line program (SLP) is a
Context-free grammar generating **exactly one** string
Two types of productions: $X_i \to a$ and $X_i \to X_p X_q$

Most of practically used compression algorithms (Lempel-Ziv family, run-length encoding...) can be efficiently translated to SLP

Example

abaababaabaab

$X_1 \to b$, $X_2 \to a$
$X_3 \to X_2X_1$, $X_4 \to X_3X_2$
$X_5 \to X_4X_3$, $X_6 \to X_5X_4$
$X_7 \to X_6X_5$



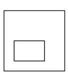
Important Related Results

Algorithms on compressed texts:

- **Amir et al.'94:** Compressed Pattern Matching
- **Gąsieniec et al.'96:** Regular Language Membership

The following problems are hard for compressed texts:

- **Lohrey'04:** Context-Free Language Membership
- **Berman et al.'02:** Two-dimensional Compressed Pattern Matching



FCPM: Problem Description

Fully Compressed Pattern Matching (FCPM)

INPUT: SLP-compression of P and of T
OUTPUT: Yes/No (whether P is a substring in T?)

Let m and n be the sizes of straight-line programs generating correspondingly P and T

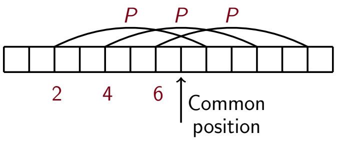
Gąsieniec et al.'96: $O((n+m)^5 log^3|T|)$ algorithm
Miyazaki et al.'97: $O(n^2m^2)$ algorithm

Lifshits'06: $O(n^2m)$ algorithm

Basic Lemma

Notation:
Position = place between neighbor characters.
Occurrence = starting position of a substring

Lemma
All occurrences of P in T touching any given position form a single arithmetical progression

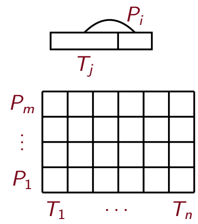


AP-table

Let $P_1, \dots, P_m$ and $T_1, \dots, T_n$ be the compression symbols.

A **cut** is a merging position for $X_i = X_r X_s$.

AP-table:
For every $1 \leq i \leq m, 1 \leq j \leq n$
let $AP[i,j]$ be a code
of ar.pr. of occurrences of P_i in T_j
that touches the cut of T_j



Two Claims

Claim 1: We can solve all variants of FCPM from AP-table in linear time:

- Find the first occurrence
- Count the number of all occurrences
- Check whether there is an occurrence from the given position
- Compute a "compressed" representation of all occurrences

Claim 2: We can compute the whole AP-table by dynamic programming method using $O(n)$ time for every element

Getting the answer

AP-table:
for every $1 \leq i \leq m, 1 \leq j \leq n$ let $AP[i,j]$ be a code
of ar.pr. of occurrences of P_i in T_j
that touches the cut of T_j

How to check whether P occurs in T from AP-table?

Answer:
P occurs in T iff there is j such that $AP[m,j]$ is nonempty

Computing AP-table

Order of computation:
from j=1 to n do
from i=1 to m do
compute AP[i,j]

Basis: one-letter P_i or one-letter T_j
Induction step: P_i and T_j are composite texts

We design a special auxiliary procedure that extracts useful information from already computed part of AP-table for computing a new element $AP[i,j]$

Auxiliary Procedure: Local PM

$LocalPM(i, j, [\alpha, \beta])$ returns occurrences of P_i in T_j inside the interval $[\alpha, \beta]$

Important properties:

- Local PM uses values AP[i,k] for $1 \leq k \leq j$
- It is defined only when $|\beta - \alpha| \leq 3|P_i|$
- It works in time $O(n)$
- The output of Local PM is a pair of ar.pr.

Proposition: answer of Local PM indeed could be always represented by pair of ar.pr.

Computing the next element

Let $P_i = P_r P_s$, and let $|P_r| \geq |P_s|$

Naive approach

1. Compute all occurrences of P_r around cut of T_j
2. Compute all occurrences of P_s around cut of T_j
3. Shift the latter by $|P_r|$ and intersect

Remark: we can do only step 1 by Local PM
Idea: not all occurrences of P_s but only these that are starting at the ends of P_r ones.

Computing the next element II

Some blackboard explanation...

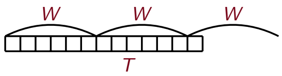
1. Take the first ar.pr of P_r occurrences
2. Divide all ends to "continental" and "seaside"
3. Check one continental
4. Check all seaside (by Local PM)
5. The same for the second ar.pr.

Total complexity:
Local PM for P_r
+ 2 Local PM for P_s
+ 2 point checks for P_s
$O(n)$

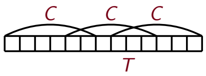
We are done! (Modulo basic computation of AP-table and realization of Local PM)

Covers and Periods

A **period** of a string T is a string W such that T is a prefix of W^k for some integer k



A **cover** of a string T is a string C such that any character in T is covered by some occurrence of C in T



Compressed Periods/Covers: given a compressed string T, to find the shortest period/cover and compute a "compressed" representation of all periods/covers

Subsequence Problems

Compressed Window Subsequence: given a pattern P, a compressed string T, and an integer k, to determine whether P is a **scattered** subsequence in some window of length k in the text T

Example

T: abaa|**babaab**|aab
P: babab
k: 6

Fully Compressed Subsequence Problem: given compressed strings P and T, to determine whether P is a **scattered** subsequence in T

Example

T: a**baababaabaab**
P: baabaabaab

Hamming Distance and LCS

Compressed Hamming Distance: given compressed strings T_1 and T_2, to compute Hamming distance (the number of characters which differ) between them

Example

T_1: **abaababa**abaab $HD(T_1, T_2) = 7$
T_2: **baabababa**baab

Compressed LCS: given compressed strings T_1 and T_2, to compute the length of the longest common subsequence

Example

T_1: a**baababaabaab** $LCS(T_1, T_2) = 12$
T_2: **baababa**ba**baab**

Fingerprint Table

A **fingerprint** is a set of used characters of any substring of T. A **fingerprint table** is the set of all fingerprints.

Example

Text: abacaba
Fingerprint Table: $\varnothing\{a\}\{b\}\{c\}\{a,b\}\{a,c\}\{a,b,c\}$

Compressed Fingerprint Table: given a compressed string T, to compute a fingerprint table

Check Your Intuition

Which of the following problems have polynomial algorithms?

1. **Periods**
2. Longest Common Subsequence
3. Hamming distance
4. **Covers**
5. **Fingerprint Table**
6. **Compressed Window Subsequence**
7. Fully Compressed Subsequence Problem

Answer: **red-on-grey** problems have polynomial algorithms, black ones are NP-hard

Summary

Main points:

- New field: algorithms working on compressed objects (including strings) without unpacking them
- New algorithm: fully compressed pattern matching in cubic time
- More algorithms: covers, periods, window subsequence, fingerprint table. But LCS, Hamming distance, FCSP are NP-hard.

Open Problems

- To construct a $O(nm \log |T|)$ algorithm for Fully Compressed Pattern Matching
- To construct $O(nm)$ algorithms for edit distance, where n is the length of T_1 and m is the **compressed size** of T_2

Last Slide

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Thanks for attention. **Questions?**